Passport to Advanced Math

Advanced Math focuses on the math you'll need to pursue further study in disciplines such as science or economics and for career opportunities in the STEM fields of science, technology, engineering, and math.

The Advanced Math area measures skills and knowledge central for progression to more advanced math courses, including demonstrating an understanding of absolute value, quadratic, exponential, polynomial, rational, radical, and other nonlinear equations. There will be 13-15 Advanced Math questions on the SAT Exam.

Advanced Math includes the following types of questions:

- Create a quadratic or exponential function or equation that models a context.
- Determine the most suitable form of an expression or equation to reveal a particular trait, given a context.
- Create equivalent expressions involving rational exponents and radicals, which include simplifying or rewriting in other forms.
- Create an equivalent form of an algebraic expression by using structure and fluency with operations.
- Solve a quadratic equation having rational coefficients. The equation can be presented in a wide range of forms to reward attending to algebraic structure and can require manipulation to solve.
- Add, subtract, and multiply polynomial expressions. Simplify the result. The expressions will have rational coefficients.
- Solve an equation in 1 variable that contains radicals or contains the variable in the denominator of a fraction.
- Solve a system of 1 linear equation and 1 quadratic equation.
- Rewrite simple rational expressions.
- Interpret parts of nonlinear expressions in terms of their context.
- Understand the relationship between zeros and factors of polynomials. Use that knowledge to sketch graphs.
- Understand a nonlinear relationship between 2 variables by making connections between their algebraic and graphical representations.
- Use function notation, and interpret statements using function notation.
- Use structure to isolate or identify a quantity of interest in an expression or isolate a quantity of interest in an equation.

TABLE OF CONTENT

Polynomial Expressions

Questions on the SAT Math exam will require you be competent in the following

(a) Adding polynomials
(b) Subtracting polynomial
(c) Multiplying & dividing polynomials
(d) Factoring Polynomials:

- **Factoring quadratic polynomials**

$$\boldsymbol{ax^2 + bx + c = (mx + p)(nx + q)}$$

- **Factoring difference of two squares**

$$\boldsymbol{a^2 - b^2 \equiv (a + b)(a - b)}$$

- **Factoring sum of cubes**

$$\boldsymbol{a^3 + b^3 \equiv (a + b)(a^2 - ab + b^2)}$$

- **Factoring difference cubes**

$$\boldsymbol{a^3 - b^3 \equiv (a - b)(a^2 + ab + b^2)}$$

- **Factoring difference of forth powers**

$$\boldsymbol{a^4 - b^4 \equiv (a^2 + b^2)(a + b)(a - b)}$$

(e) Factoring higher-degree polynomial
(f) Simplifying rational expression by factoring.

Binomial Squares

$$(p + q)^2 = p^2 + q^2 + 2pq$$

$$(p - q)^2 = p^2 + q^2 - 2pq$$

$$\boldsymbol{p^2 + q^2 = (p + q)^2 - 2pq}$$

$$\boldsymbol{p^2 + q^2 = (p - q)^2 + 2pq}$$

$$2(p^2 + q^2) = (p + q)^2 + (p - q)^2$$

$$\boldsymbol{p^2 + q^2 = \frac{(p + q)^2 + (p - q)^2}{2}}$$

FACTORS

Consider $f(x) = x^2 + 5x + 6$

factorized $f(x) = (x + 2)(x + 3)$

which means $x + 2$ and $x + 3$ are factors of $f(x) = x^2 + 5x + 6$. Since $x + 2$ is a factor of $f(x) = x^2 + 5x + 6$

then $$f(-2) = (-2)^2 + 5(-2) + 6$$

$$f(-2) = 4 - 10 + 6$$

$$f(-2) = 0$$

Likewise

$$f(-3) = (-3)^2 + 5(-3) + 6$$

$$f(-3) = 9 - 15 + 6$$

$$f(-3) = 0$$

The above is referred to as the Factor Theorem.

Factor Theorem

- If $f(a) = 0$, then $f(x)$ has a factor $(x - a)$
- If $(x + a)$ is a factor of $f(x)$, then $f(-a) = 0$

EXERCISE

Given the function $h(x) = 2x^3 - 3x^2 - 7x + 10$, and $h(2) = 0$, then $(x - 2)$ is factor.

$$(\boldsymbol{x - 2})(2x^2 + x + 5)$$

LONG DIVISION OF POLYNOMIALS

EXERCISE

$$g(x) = 3x^2 + 17x + 23$$

What is the remainder when the above function is divided by $x + 2$

$$\begin{array}{r} 3x+11 \\ x+2\overline{)\,3x^2+17x+23} \\ -\underline{(3x^2+6x)} \\ 11x+23 \\ -\underline{(11x+22)} \\ 1 \end{array}$$

Remainder

Another way to write the same information:

$$\frac{3x^2+17x+23}{x+2} = 3x+11+\frac{1}{x+2}$$

When $g(x) = 3x^2 + 17x + 23$ is divided by $x + 2$ the remainder is 1

$$\begin{aligned} g(-2) &= 3(-2)^2 + 17(-2) + 23 \\ &= 12 - 34 + 23 \\ &= 35 - 34 \\ g(-2) &= 1 \end{aligned}$$

The above is referred to as the Remainder Theorem

When polynomial $f(x)$ is divided by $(x - a)$, the remainder R is equal to $f(a)$
Polynomial $f(x)$ can be expresses as follows.

$$f(x) = (x - a)Q(x) + R$$

Where $Q(x)$ is the Quotient and R is the remainder.

1.

Which expression is equivalent to $x^4 - 18x^2 + 81$?

A) $(x-3)^4$

B) $(x-3)(x+3)^3$

C) $(x-3)^3(x+3)$

D) $(x-3)^2(x+3)^2$

2.

Which expression is equivalent to

$$\frac{y+3}{x-1} + \frac{y(x-1)}{x^2y - xy}$$

A) $\frac{xy^2+4xy-y}{x^2y-xy}$

B) $\frac{xy^2+4xy-y}{x^3y-2xy-xy}$

C) $\frac{xy^2+y+2}{x^3y-2x^2y-xy}$

D) $\frac{xy^2+2xy+3}{x^2y-xy+x-1}$

3.

$$\frac{(x-\sqrt{3})(x+\sqrt{3})}{3(x^2-3)}$$

Which of the following is equivalent to the give expression for values of x not equal to $\sqrt{3}$ or $-\sqrt{3}$?

A) $\sqrt{3}$

B) $\frac{\sqrt{3}}{3}$

C) $\frac{2}{3}$

D) $\frac{1}{3}$

4.

Which expression is equivalent to $6x^4 + 7x^2 + 2$

A) $x^2(6x+7)+2$

B) $x^2(6x^2+7x+2)$

C) $(3x+2)(2x+1)$

D) $(3x^2+2)(2x^2+1)$